

Analysis of Generalized Ghost Version of Pilgrim Dark Energy

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Abstract

The proposal of pilgrim dark energy is based on the speculation that phantom-like dark energy possesses enough resistive force to preclude the black hole formation in the later universe. We explore this phenomenon by assuming the generalized ghost version of pilgrim dark energy. We find that most of the values of the interacting (ξ^2) as well as pilgrim dark energy (u) parameters push the equation of state parameter towards phantom region. The squared speed of sound shows that this model remains stable in most of the cases of ξ^2 and u . We also develop $\omega_\Lambda - \omega'_\Lambda$ plane and observe that this model corresponds to thawing as well as freezing regions. Finally, it is shown that the non-interacting and interacting generalized ghost versions of pilgrim dark energy correspond to Λ CDM limit on the statefinder plane.

Keywords: Pilgrim dark energy; Cold dark matter; Cosmological parameters.

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1 Introduction

The observational analysis through different schemes indicates that the universe undergoes an accelerated expansion. The first clue about this expansionary phenomenon was given by a variety of astronomers through Supernova type Ia (Riess et al. 1998; Perlmutter et al. 1999) and subsequent attempts also favor this phenomenon (Caldwell and Doran 2004; Koivisto and Mota 2006; Hoekstra and Jain 2008). This phenomenon is assumed to be taken place under the influence of an exotic type force termed as dark energy (DE) whose nature is still unknown. To peruse this problem, a versatile study has been done in various ways which has led many dynamical DE models (Armendariz-Picon et al. 1999; Caldwell 2002; Bagla et al. 2003; Hsu 2004; Li 2004; Feng et al. 2005; Zhang et al. 2006; Cai 2007) and modified theories of gravity (Brans and Dicke 1961; Linder 2010; Dutta and Saridakis 2010; Sharif and Rani 2013a; 2013b).

The dynamical DE models have been constructed in two frameworks: quantum gravity and general relativity. The holographic dark energy (HDE) model (Li 2004) has been developed in the context of quantum gravity with the help of holographic principle (Susskind 1995). The basic idea behind this model is that the bound on the vacuum energy (Λ) of a system with size L should not cross the limit of the BH mass having the same size due to the formation of BH in quantum field theory (Cohen et al. 1999). Wei (2012) reconsidered this idea and suggested that if we are able to prevent the BH formation in the later universe then the energy bound proposed by Cohen et al. (1999) could be violated. For this purpose, the strong repulsive force is required which may help in the avoidance of matter collapse and hence the BH formation.

The question arises which type of force can play this role? In this regard, phantom-like DE can give useful contribution as compared to its other versions (vacuum and quintessence-like). It is well-known that due to phantom-like DE, everything will be crashed before our universe ends in the big-rip and ultimately BH formation would be avoided in this way. The effects of phantom-like DE have been explored in different ways. Babichev et al. (2004) suggested that when phantom-like DE accreted onto a BH, it leads to the loss of BH mass with the passage of time. This phenomenon has been tested through different phantom-like family of chaplygin gas models which lead to similar results (Martin-Moruno 2008; Jamil et al. 2008; Babichev et al. 2008; Jamil 2009; Jamil and Qadir 2011; Sharif and Abbas 2011; Bhadra

and Debnath 2012). In the wormhole physics, phantom-like DE plays an effective role in precluding the formation of event horizon (Lobo 2005a, 2005b; Sushkov 2005, Sharif and Jawad 2014).

Harada et al. (2006) argued that self-similar solution does not exist for a universe filled with quintessence or scalar field or stiff fluid. The same results have been found by using different approaches (Akhoury et al. 2009). However, in these works, only the strong energy condition is violated which is inferred as quintessence-like DE and does not possess enough resistive force to preclude the BH formation. It was also pointed out by Li and Wang (2007) that BHs can exist in FRW universe in the presence of DE which only satisfies weak energy condition. Recently, Wei (2012) has suggested that phantom-like DE is more effective since it violates all the energy conditions. He constructed the following model

$$\rho_\Lambda = 3\delta^2 m_p^{4-u} L^{-u}, \quad (1)$$

where δ and u appear as dimensionless constants, L is known as infrared cutoff and m_p is the reduced Planck constant. This model is called pilgrim DE (PDE). He considered PDE with Hubble horizon as an IR cutoff and pointed out (through theoretical and observational ways) different possibilities for the avoidance of BH in the later universe.

We have extended this work for non-interacting and interacting PDE with different IR cutoffs in the flat as well as non-flat universe models (Sharif and Jawad 2013a, 2013b). In these works, we have found that EoS parameter for non-interacting and interacting cases lies in the phantom region for $u > 0$ as well as $u < 0$ which favor PDE phenomenon. We have also developed $\omega_\Lambda - \omega'_\Lambda$ and $r - s$ planes for these models. Here, we explore the non-interacting and interacting generalized ghost PDE models in the flat universe. We analyze the behavior of EoS parameter, squared speed of sound (for stability), $\omega_\Lambda - \omega'_\Lambda$ and $r - s$ planes. The paper is organized as follows. Section 2 is devoted to the basic equations, EoS parameter and the stability analysis. In sections 3 and 4, we present the analysis of $\omega_\Lambda - \omega'_\Lambda$ and $r - s$ planes, respectively. We summarize our results in the last section.

2 Equation of State Parameter

In this section, we present basic scenario of interacting generalized ghost DE version of PDE with cold dark matter (CDM) in flat universe. We

extract equation of state parameter and analyze its behavior through PDE parameter. The first equation of motion corresponding to flat universe leads to

$$H^2 = \frac{1}{3m_{pl}^2}(\rho_m + \rho_\Lambda), \quad (2)$$

where ρ_m and ρ_Λ indicate CDM and DE densities. In terms of fractional energy density, this equation becomes

$$\Omega_m + \Omega_\Lambda = 1, \quad \Omega_m = \frac{\rho_m}{3m_{pl}^2 H^2}, \quad \Omega_\Lambda = \frac{\rho_\Lambda}{3m_{pl}^2 H^2}. \quad (3)$$

The proposal of Veneziano ghost DE ($\rho_\Lambda = \alpha H$, where α is a constant with dimension $[energy]^3$) lies in the category of dynamical DE models which plays an important role in the accelerated expansion of the universe (Urban and Zhitnitsky 2009a, 2009b, 2010a, 2010b, 2011). The motivation of this model comes from Veneziano ghost of chromodynamics (QCD) which is useful to solve $U(1)$ problem in QCD. The key feature of this model is that Veneziano ghost (being unphysical in quantum field theory formulation in the Minkowski spacetime) provides non-trivial physical effects in FRW universe (Rosenzweig et al. 1980; Nath and Arnowitt 1981). The QCD ghost has a little contribution in the vacuum energy density proportional to $\Lambda_{QCD}^3 H$ (where $\Lambda_{QCD} \sim 100 MeV$ is the smallest QCD scale and H represents the Hubble parameter), but this contribution is very crucial in the evolutionary behavior of the universe. This model helps in alleviating the fine tuning as well as cosmic coincidence problem (Urban and Zhitnitsky 2009a, 2009b, 2010a, 2010b, 2011; Forbes and Zhitnitsky 2008). Several theoretical aspects have been investigated for this model (Ebrahimi and Sheykhi 2011; Sheykhi and Sadegh 2012; Sheykhi and Bagheri 2011; Rozas-Fernandez 2012; Karami and Fahimi 2013) and tested thorough different observational schemes (Cai et al. 2011).

It is noted that vacuum energy from Veneziano ghost field in QCD is of the form $H + O(H^2)$ (Zhitnitsky 2012), but in the ordinary ghost DE model, only the leading term (i.e., H) has been considered. Cai et al. (2012) suggested that the contribution of subleading term (i.e., H^2) in the ordinary ghost DE can be helpful in describing the early evolution of the universe. He proposed the so called generalized ghost DE density

$$\rho_\Lambda = \alpha H + \beta H^2,$$

where β is another constant with dimension $[energy]^2$. Theoretically, different cosmological parameters such as EoS parameter, deceleration, $\omega_\Lambda - \omega'_\Lambda$ and statefinders etc have been developed for this model (Ebrahimi et al. 2012; Malekjani 2013; Karami 2013). The stability of this model has also been investigated (Ebrahimi and Sheykhi 2013). Here, we use this model in order to discuss the PDE phenomenon. In terms of PDE, the generalized ghost DE density takes the form

$$\rho_\Lambda = (\alpha H + \beta H^2)^u, \quad (4)$$

known as generalized ghost PDE.

We take interaction between generalized ghost version of PDE with CDM which follows the equations of continuity as

$$\dot{\rho}_m + 3H\rho_m = \Delta, \quad \dot{\rho}_\Lambda + 3H(\rho_\Lambda + p_\Lambda) = -\Delta, \quad (5)$$

where Δ possesses dynamical nature and appears as interaction term between CDM and generalized ghost DE version of PDE. In general, three forms are commonly used given as follows

$$\begin{aligned} \Delta_1 &= 3\xi^2 H \rho_\Lambda, \\ \Delta_2 &= 3\xi^2 H (\rho_m + \rho_\Lambda), \\ \Delta_3 &= 3\xi^2 H \rho_m. \end{aligned}$$

Here ξ^2 is the coupling constant whose sign is important in the present cosmological evolution of the universe. The positive ξ^2 represents the decay of DE into DM while negative ξ^2 is responsible for decomposition of DM into DE. It is argued (Pavon and Wang 2009) that the sign of ξ^2 should be taken as positive according to thermodynamical view point as second law of thermodynamics favors the decay of DE into DM.

Moreover, it was found that observations also support the phenomenon of decaying of CDM into DE (Pereira and Jesus 2009; Guo, et al. 2007; Costa, et al. 2009). Cai and Su (2010) fitted the interaction term Δ with observations without choosing any specific form of it. They found that Δ crosses the non-interacting line ($\Delta = 0$) and changes its sign around $z = 0.5$. This behavior of interacting term raises remarkable challenge to the interacting models since the above three forms of interaction do not change their signs during the cosmological evolution. This gives a clue to the proposal of most general form of interaction.

This is why, Sun and Yue (2012) to proposed new form of interaction term as follows

$$\Delta_4 = 3\xi^2 H(\rho_\Lambda - \rho_m), \quad (6)$$

Here $\xi^2 > 0$ because its negative value conducts the matter energy density to be negative. In the early universe, $\rho_m > \rho_\Lambda$ leads to $\Delta_4 < 0$ while in present epoch, it changes its sign after the transition of universe from decelerated to accelerated regime. As, our universe shows transition from deceleration to acceleration at the redshift value $z = 0.5$. It is also found that this interaction term shows the compatibly with the universe transition at $z = 0.5$. Also, the scenario corresponding to the above interaction term satisfied the generalized second law of thermodynamics. They have also compared this interacting model with observational data and found its consistencies with the result in (Daly et al. 2008) and the 7-years WMAP observations (Komatsu et al. 2011).

Differentiating Eq.(4) with respect to $x = \ln a$, we obtain

$$\rho'_\Lambda = u\rho_\Lambda \left(\frac{\alpha + 2\beta H}{\alpha + \beta H} \right) \frac{\dot{H}}{H^2}. \quad (7)$$

where prime means differentiation with respect to x . Using Eq.(6)-(7), we obtain the EoS parameter as

$$\omega_\Lambda = -1 - \xi^2 \left(1 - \frac{\Omega_m}{\Omega_\Lambda} \right) - \frac{u}{3} \left(\frac{\alpha + 2\beta H}{\alpha + \beta H} \right) \frac{\dot{H}}{H^2}. \quad (8)$$

In order to eliminate the term $\frac{\dot{H}}{H^2}$ from this equation, we use Eqs.(2)-(7)

$$\frac{\dot{H}}{H^2} = \frac{(3\xi^2\Omega_\Lambda - \Omega_m(3\xi^2 + 1))(\alpha + \beta H)}{2(\alpha + \beta H) - u\Omega_\Lambda(\alpha + 2\beta H)}. \quad (9)$$

Thus the EoS parameter becomes

$$\omega_\Lambda = -1 - \xi^2 \left(1 - \frac{\Omega_m}{\Omega_\Lambda} \right) - \frac{u(\alpha + 2\beta)(3\xi^2\Omega_\Lambda - \Omega_m(3\xi^2 + 1))}{3(2(\alpha + \beta H) - u\Omega_\Lambda(\alpha + 2\beta H))}. \quad (10)$$

We plot this EoS parameter versus PDE parameter u for its three different ranges $0 < u \leq 1.3$, $1.4 \leq u \leq 12$ and $-16 \leq u \leq 0$ as shown in Figures **1-3**. To analyze the behavior of EoS parameter more clearly, we choose three different well-known values of interacting parameter $\xi^2 = 0, 0.5, 1$ and

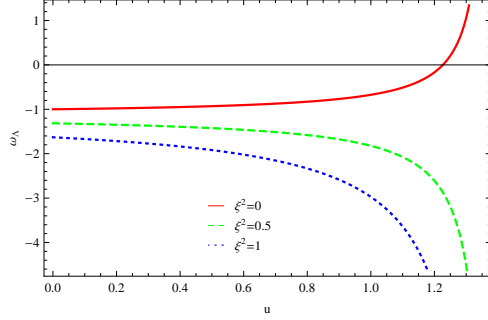


Figure 1: Plot of ω_Λ versus u for generalized ghost version of PDE with $0 \leq u \leq 1.3$.

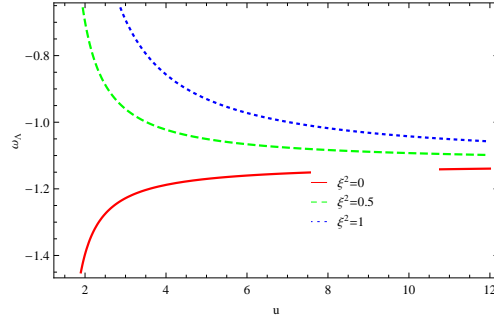


Figure 2: ω_Λ versus u for generalized ghost version of PDE with $u \geq 1.4$.

keep the present values of other parameters such as $\Omega_\Lambda = 0.76$, $\Omega_m = 0.24$ (Suyu et al. 2013). Also, the values of the generalized ghost version of PDE parameters are $\alpha = 1.55$, $\beta = 1.91$. In Figure 1 (for $\xi^2 = 0$), the EoS parameter meets Λ CDM model for $0 < u \leq 0.4$, maintains the quintessence DE region for $0.4 < u \leq 1.2$ and then goes towards DM region of the universe. Consequently, this model does not favor the phenomenon of PDE. For $\xi^2 = 0.5, 1$, the EoS parameter starts from $-1.25, -1.6$ and goes towards more negative values. Thus EoS parameter indicates the presence of less phantom energy at low values of u and converges towards strong phantom region with the increase of u . This behavior strongly favors the PDE phenomenon and also there is a possibility of big-rip singularity.

In Figure 2, it is observed that the present values of EoS parameter start from the high phantom region and approach to less phantom region, i.e., $\omega_\Lambda = -1.15$ for $\xi^2 = 0$. When $\xi^2 = 0.5$, ω_Λ represents quintessence region

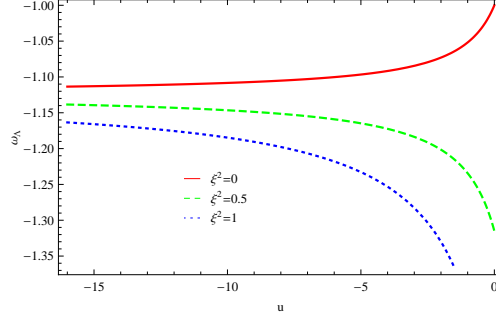


Figure 3: ω_Λ versus u with $u \leq 0$.

in the range $1.8 \leq u < 3.25$, represents the Λ CDM model at $u = 3.25$ and approaches to phantom era value -1.1 for $u > 3.25$. In case of $\xi^2 = 1$, the present values of ω_Λ start from quintessence region and approach to phantom region by crossing the Λ CDM limit. In this graph, all three models correspond to phantom region which is useful for PDE prediction. In Figure 3, the model with $\xi^2 = 0$ coincides with Λ CDM limit at $u = 0$ and goes towards phantom region for $u < 0$. For $\xi^2 = 0.5, 1$, the present values of EoS parameter lie in the phantom region for $u \leq 0$.

Now, we use squared speed of sound for the stability analysis of the present interacting model given by

$$v_s^2 = \frac{\dot{p}}{\dot{\rho}} = \frac{p'}{\rho'}, \quad (11)$$

Using Eqs.(4), (6), (7), (9) and (11), it follows that

$$\begin{aligned} v_s^2 = & \frac{1}{3}(-3 + 3\xi^2(-2 + \Omega_\Lambda^{-1}) + (u(-1 - 3\xi^2 + \Omega_\Lambda + 6\xi^2\Omega_\Lambda)(\alpha \\ & + 2\beta H))(-2(\alpha + \beta H) + u\Omega_\Lambda(\alpha + 2\beta H))^{-1} + ((\alpha + \beta H)(2(\alpha \\ & + \beta H) - u\Omega_\Lambda H(\alpha + 2\beta H))(((-1 + u)(12\xi^2(\alpha + \beta H)^2 - 12\xi^2 u\Omega_\Lambda \\ & \times (\alpha + \beta H)(\alpha + 2\beta H) + u\Omega_\Lambda^2(\alpha + 2\beta H)(\alpha(2 + 12\xi^2 - u) + 2\beta(1 \\ & + 6\xi^2 - u)H)))(-2(\alpha + \beta H) + u\Omega_\Lambda(\alpha + 2\beta H))^{-1} + (2\alpha\beta u\Omega_\Lambda \\ & \times (-1 - 3\xi^2 + \Omega_\Lambda + 6\xi^2\Omega_\Lambda)H)(-2\alpha + H(-2\beta + u\Omega_\Lambda(\alpha + 2\beta H)))^{-1})) \\ & \times (u\Omega_\Lambda(\alpha + 2\beta H)(-2(\alpha + \beta H) + u\Omega_\Lambda(\alpha + 2\beta H))^2)^{-1}). \end{aligned}$$

The behavior of v_s^2 against PDE parameter u for its three ranges (keeping other cosmological parameters the same) is shown in Figures 4-6. In Figure

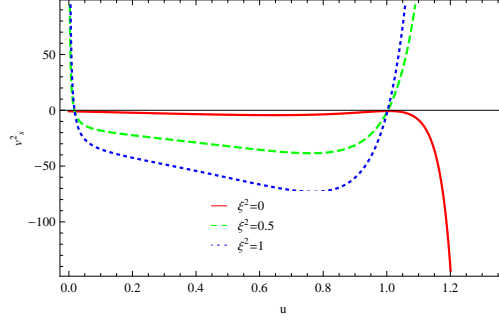


Figure 4: v_s^2 versus u for generalized ghost version of PDE with $0 \leq u \leq 1.3$.

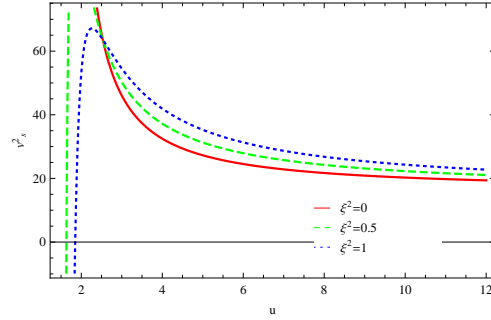


Figure 5: v_s^2 versus u for generalized ghost version of PDE with $u \geq 1.4$.

4, the non-interacting generalized ghost version of PDE remains stable in the ranges $0 \leq u \leq 0.25$ and $0.9 \leq u \leq 1.1$ while exhibits instability in the ranges $0.25 < u < 0.9$ and $u > 1.1$. However, the models corresponding to interacting cases $\xi^2 = 0.5, 1$ show stability in the ranges $0 \leq u \leq 0.02, u \geq 0.95$ and instability for $0.02 < u < 0.95$. The squared speed of sound remains positive for non-interacting case as shown in Figure 5. However, its behavior is similar for interacting cases, i.e, it starts from negative values (represents instability of the models), goes towards positive maxima and eventually decreases and approaches to positive value (exhibits stability of the models). It is remarked that these models remain stable forever for $u \geq 2.5, u > 1.6, u > 1.8$ corresponding to $\xi^2 = 0, 0.5, 1$ cases, respectively. Also, it is observed that $v_s^2 < 0$ for $-0.1 < u < 0, -0.8 < u < 0, -1.5 < u < 0$ for $\xi^2 = 0, 0.5, 1$, respectively which shows that these models exhibit instability for these ranges of u as shown in Figure 6. However, these models corresponding to $\xi^2 = 0, 0.5, 1$ remain stable for $u \leq -0.1, -0.8, -1.5$, respectively.

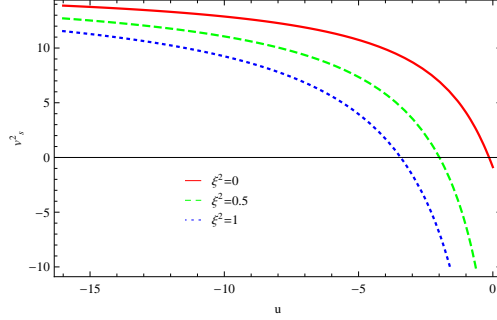


Figure 6: v_s^2 versus u with $u \leq 0$.

3 $\omega_\Lambda - \omega'_\Lambda$ Analysis

The $\omega_\Lambda - \omega'_\Lambda$ plane, firstly proposed by Caldwell and Linder (2005), has become useful tool for distinguishing different DE models through trajectories on its plane. Initially, this approach has been applied on quintessence DE model which leads to two classes of its plane, i.e, the area occupied by the region ($\omega'_\Lambda > 0$, $\omega_\Lambda < 0$) on $\omega_\Lambda - \omega'_\Lambda$ plane corresponds to thawing region while area under the region ($\omega'_\Lambda < 0$, $\omega_\Lambda < 0$) implies the freezing region. It is observed that the expansion of the universe is comparatively more accelerating in freezing region. Later, this tool has been applied to other well-known dynamical DE models such as more general form of quintessence (Scherrer 2006), phantom (Chiba 2006), quintom (Guo et al. 2006), polytropic DE (Malekjani and Khodam-Mohammadi 2012) and PDE (Sharif and Jawad 2013a, 2013b) (in flat and non-flat universes) models. Here we use this analysis to explore these regions. Differentiating Ω_Λ with respect to x and using Eqs.(2)-(4) and (9), we obtain

$$\begin{aligned} \Omega'_\Lambda &= \Omega_\Lambda [u(\alpha + 2\beta H) - \alpha - \beta H] [-\Omega_m + 3\xi^2(\Omega_\Lambda - \Omega_m)] [2(\alpha \\ &+ 2\beta H) - u\Omega_\Lambda(\alpha + 2\beta H)]^{-1}, \end{aligned} \quad (12)$$

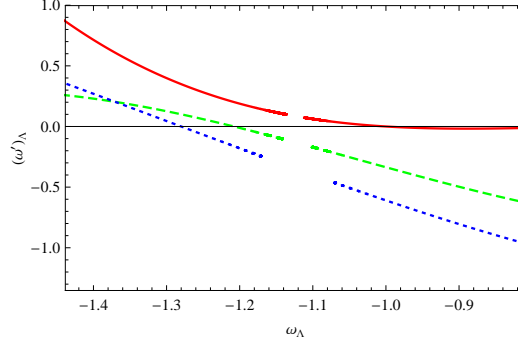


Figure 7: Plot of $\omega_\Lambda - \omega'_\Lambda$ for generalized ghost version of PDE. Red, green and blue lines correspond to $\xi^2 = 0, 0.5, 1$, respectively.

By taking the derivative of Eq.(10) and using the above expression, we get the evolutionary form of ω_Λ as follows

$$\begin{aligned} \omega'_\Lambda = & -((2\alpha\beta u\Omega_\Lambda^2(-1 - 3\xi^2 + \Omega_\Lambda + 6\xi^2\Omega_\Lambda)(-1 + \Omega_\Lambda + 3\xi^2(-1 \\ & + 2\Omega_\Lambda))H(\alpha + \beta H))(2(\alpha + \beta H) - u\Omega_\Lambda H(\alpha + 2\beta H))^{-1}) - (\Omega_\Lambda \\ & \times (-1 + \Omega_\Lambda + 3\xi^2(-1 + 2\Omega_\Lambda))(-\alpha - \beta H + u(\alpha + \beta H))(12\xi^2(\alpha \\ & + \beta H)^2 - 12\xi^2 u\Omega_\Lambda(\alpha + \beta H)(\alpha + 2\beta H) + u\Omega_\Lambda^2(\alpha + 2\beta H) \\ & \times (\alpha(2 + 12\xi^2 - u) + 2\beta(1 + 6\xi^2 - u)H)))(2(\alpha + \beta H) - u\Omega_\Lambda \\ & \times (\alpha + 2\beta H))^{-1})(3\Omega_\Lambda^2(-2(\alpha + \beta H) + u\Omega_\Lambda(\alpha + 2\beta H))^2)^{-1}. \end{aligned} \quad (13)$$

We can obtain the $\omega_\Lambda - \omega'_\Lambda$ plane by plotting ω'_Λ versus ω_Λ and keeping the same values of constant cosmological parameters as shown in Figure 7. It can be observed that Λ CDM limit ($\omega'_\Lambda = 0$ when $\omega_\Lambda = -1$) can be achieved for non-interacting case. Also, the curve corresponds to the crossing line of freezing and thawing regions, i.e., $\omega'_\Lambda = 0$ upto the range $-1.02 \leq \omega_\Lambda \leq -0.8$ while it goes towards thawing region for $\omega_\Lambda \leq -1.02$. For interacting case $\xi^2 = 0.5$, the curve does not meet the Λ CDM limit. However, it remains in the freezing region in the range $-1.2 < \omega_\Lambda \leq -0.8$ and in the thawing region for $\omega_\Lambda < -1.2$. When $\omega_\Lambda = -1.2, -1$, ω'_Λ approaches to 0 and -0.3 , respectively. For interacting case $\xi^2 = 1$, the freezing and thawing regions are also obtained in the ranges $\omega_\Lambda < -1.28$ and $-1.28 < \omega_\Lambda \leq -0.8$, respectively while for $\omega_\Lambda = -1.28, -1$, ω'_Λ approaches to 0 and -0.6 , respectively.

4 Statefinder Parameters

Since now, a variety of DE models have been proposed for explaining the accelerated expansion phenomenon of the universe. In order to check the viability of these models, statefinder parameters are widely used (Sharif and Jawad 2013a, 2013b; Sahni et al. 2003; Alam et al. 2003; Feng 2008; Setare et al. 2007; Malekjani et al. 2011). The cosmological plane corresponding to these parameters termed as $r-s$ plane and the trajectories tell the distance of a given DE model from Λ CDM limit. The statefinder parameters for flat universe are defined as follows

$$r = \frac{\ddot{a}}{aH^3}, \quad s = \frac{r-1}{3(q-\frac{1}{2})}, \quad (14)$$

where q indicates the deceleration parameter. The cosmological plane of these parameters describe different well-known regions of the universe, i.e., $s > 0$ and $r < 1$ describe the region of phantom and quintessence DE eras, $(r, s) = (1, 0)$ corresponds to Λ CDM limit, $(r, s) = (1, 1)$ represents CDM limit and $s < 0$ and $r > 1$ indicates chaplygin gas. Using Eqs.(9) and (14), it follows that

$$\begin{aligned} r &= 1 - ((-1 + \Omega_\Lambda + \xi^2(-3 + 6\Omega_\Lambda))(\alpha + 2\beta H)u(-6(\Omega_\Lambda + \xi^2(-1 \\ &+ 2\Omega_\Lambda))(\alpha + \beta H) + \Omega_\Lambda(1 + 2\Omega_\Lambda)(\alpha + 2\beta H)u))(2(-2(\alpha + \beta H) \\ &+ \Omega_\Lambda(\alpha + 2\beta H)u)^2)^{-1} - ((-1 + \Omega_\Lambda + \xi^2(-3 + 6\Omega_\Lambda))(\alpha + \beta H) \\ &\times ((2\alpha\Omega_\Lambda(-1 + \Omega_\Lambda + \xi^2(-3 + 6\Omega_\Lambda))\beta H)u)(-2(\alpha + \beta H) + \Omega_\Lambda H \\ &\times (\alpha + 2\beta H)u)^{-1} + ((-1 + u)(12\xi^2(\alpha + \beta H)^2 + 2\Omega_\Lambda(6\xi^2(-1 \\ &+ \Omega_\Lambda) + \Omega_\Lambda)(\alpha + \beta H)(\alpha + 2\beta H)u - \Omega_\Lambda^2(\alpha + 2\beta H)^2u^2))(-2(\alpha \\ &+ \beta H) + \Omega_\Lambda(\alpha + 2\beta H)u)^{-1}))(2(-2(\alpha + \beta H) + \Omega_\Lambda(\alpha + 2\beta H)u)^2)^{-1}, \\ s &= ((-1 + \Omega_\Lambda + \xi^2(-3 + 6\Omega_\Lambda))(9(\alpha + 2\beta H)u - ((\alpha + \beta H)(6(\alpha \\ &+ \beta H) - 3\Omega_\Lambda(\alpha + 2\beta H)u)^2((2\alpha\Omega_\Lambda(1 + \Omega_\Lambda + \xi^2(-3 + 6\Omega_\Lambda)) \\ &\times \beta H)u)(-2(\alpha + \beta H) + \Omega_\Lambda H(\alpha + 2\beta H)u)^{-1} + ((-1 + u)(12\xi^2 \\ &\times (\alpha + \beta H)^2 + 2\Omega_\Lambda(6\xi^2(-1 + \Omega_\Lambda) + \Omega_\Lambda)(\alpha + \beta H)(\alpha + 2\beta H)u \\ &- \Omega_\Lambda^2(\alpha + 2\beta H)^2u^2))(-2(\alpha + \beta H) + \Omega_\Lambda(\alpha + 2\beta H)u)^{-1}))((-2(\alpha \\ &+ \beta H) + \Omega_\Lambda(\alpha + 2\beta H)u)^2(6(\Omega_\Lambda + \xi^2(-1 + 2\Omega_\Lambda))(\alpha + \beta H) \\ &- \Omega_\Lambda(1 + 2\Omega_\Lambda)(\alpha + 2\beta H)u)^{-1}))(9(-6(\alpha + \beta H) + 3\Omega_\Lambda(\alpha \\ &+ 2\beta H)u))^{-1}. \end{aligned} \quad (15)$$

$$(16)$$

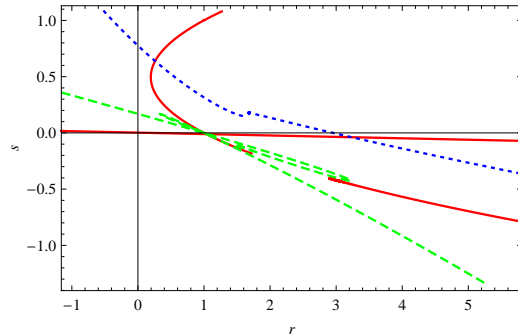


Figure 8: Plot of $r - s$ for generalized ghost version of PDE. Red, green and blue lines correspond to $\xi^2 = 0, 0.5, 1$, respectively.

We can obtain the plane of statefinders by plotting s versus r for three different choices of interacting parameter as shown in Figure 8. It can be observed from the $r - s$ plane that non-interacting and interacting ($\xi^2 = 0.5$) generalized ghost PDE models correspond to Λ CDM model. The $r - s$ plane corresponding to three different choices of interacting parameter ($\xi^2 = 0, 0.5, 1$) also provide the regions of DE (phantom and quintessence) and chaplygin gas model. In non-interacting case, the trajectory also corresponds to CDM limit.

5 Concluding Remarks

The generalized ghost DE model has been used for different purposes such as the evolution of the universe by extracting different cosmological parameters, thermodynamics laws, correspondence with different scalar field models, analysis of $\omega_\Lambda - \omega'_\Lambda$ and $r - s$ planes. Also, various cosmological aspects of this model have been addressed in different modified theories. This idea of PDE is interesting as it indicates one of the notions about the universe (phantom energy) in the later time. In this work, we have developed generalized ghost version of PDE to explain the fate of BH in the presence of large amount of phantom energy in the universe.

We have considered interacting scenario of the generalized ghost DE with CDM and evaluated two cosmological parameters (i.e., EoS and squared speed of sound) as well as two cosmological planes (i.e., $\omega_\Lambda - \omega'_\Lambda$ and $r - s$).

We have analyzed the behavior of these parameters through two constant parameters such as interacting (ξ^2) and PDE (u). We have explored EoS parameter versus PDE parameter for three different well-known values of $\xi^2 = 0, 0.5, 1$. Also, we have taken into account three ranges of PDE parameter, i.e., $0 \leq u \leq 1.3$, $u \geq 1.4$ and $u \leq 0$ as shown in Figure **1-3**. It can be observed from Figure **1** that the present values of ω_Λ do not lie in the phantom region of the universe for non-interacting case and hence it does not favor the PDE conjecture. In the interacting case, the present values of ω_Λ correspond to the phantom region and attain more negative (phantom) values with the increase of u . However, the presence of phantom energy for the interacting case ($\xi^2 = 1$) is comparatively larger than the interacting case ($\xi^2 = 0.5$).

Figure **2** shows that all the present values of EoS parameter in the non-interacting case lie in the phantom region and it attain the region of high phantom energy at $u = 1.4$. Moreover, the present values of ω_Λ approach to phantom region for $u \geq 3$ and $u \geq 8$ in the interacting cases $\xi^2 = 0.5, 1$, respectively. For $u < 0$ (Figure **3**), the trajectory of ω_Λ corresponding to all the cases of interacting parameter lies in the phantom region. However, the interacting model with $\xi^2 = 1$ acquires more phantom energy as compared to other cases. For the stability analysis of these models, we have plotted the squared speed of sound versus u (with three different ranges) for the non-interacting and interacting cases as shown in Figures **4-6**. It is found that the generalized ghost PDE exhibits stability in the ranges $u \leq -0.1$, $0 \leq u \leq 0.25$, $0.9 \leq u \leq 1.1$ and $u \geq 2.5$ for non-interacting case. For interacting case ($\xi^2 = 0.5$), this PDE model remains stable in the ranges $0 \leq u \leq 0.02$, $u \geq 1.6$ and $u \leq -0.8$. For $\xi^2 = 1$, stability of the model is observed when $u > 1.1$, $u > 1.8$ and $u \leq -1.5$.

We have also examined the consequences of $\omega_\Lambda - \omega'_\Lambda$ plane for this PDE model which shows that the Λ CDM limit is achieved only for non-interacting case. We have also mentioned different ranges of PDE parameter u , where the $\omega_\Lambda - \omega'_\Lambda$ plane corresponds to thawing and freezing regions for non-interacting as well as interacting case. Finally, we have developed $r - s$ plane and found that the trajectories corresponding to non-interacting and interacting ($\xi^2 = 0.5$) cases meet the Λ CDM limit. It is also pointed out that the $r - s$ plane for non-interacting and interacting cases possess the regions of chaplygin gas, quintessence and phantom models. We conclude that this work favors the PDE phenomenon.

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